

# Choice Reversals across Certainty, Uncertainty and Risk: The Equate-to-Differentiate Interpretation

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**Abstract** A generalized *weak dominance* approach is used to test choice reversals across certainty, uncertainty and risk. In the case of pairwise choice where each alternative is generally better than the other on a single dimension, this approach models much human choice behavior as a process in which people seek to equate smaller difference between alternatives on one dimension, thus leaving the greater one-dimensional difference to be differentiated as the determinant of the final choice. The choice reversals are therefore seen as a consequence of the fact that what is seen as the greatest one-dimensional difference on one trial is no longer seen as the greatest on another trial. A "matching" task was designed to examine whether the knowledge of the value difference of the paired outcomes along each dimension will permit prediction of preferential choice. The overall test-retest results for various choosing tasks favor the equate-to-differentiate explanation. The finding supports the claim that the repeated choices can be consistent not because the chosen alternative is always of the greatest overall worth but because final choice is consistently based on a single fixed dimension on each trial.

**Key words** repeated choices, choice reversals, weak dominance

## 1 Introduction

A widespread characteristic of human choice is that people are not perfectly consistent in their choices. When faced with repeated choices among alternatives, people often reverse their choices. It is difficult to conceive of a model which could provide a systematic account of such inconsistencies or variability in choice behavior. Choice reversals are predicted as a by-product of gaps in the choice rule by some authors (e.g., Butler<sup>[1]</sup>). To accommodate these reversals, many theorists treat choice variability as "errors of judgment" or "lapses of attention" and essentially ignore them. Some theorists hypothesize that choices should be defined in a probabilistic fashion (For detailed discussion about the probabilistic properties of choice models see Luce & Suppes<sup>[2]</sup>; Tversky<sup>[3]</sup>; and Tversky & Russo<sup>[4]</sup>). On each trial, participants state their choice. A stochastic choice of alternative  $i$  over alternative  $j$  is then said to occur when  $P(i, j)$ , the proportion of time  $i$  is chosen over  $j$ , exceeds 0.5.

As an alternative approach to human decision making, the equate-to-differentiate model<sup>[5~9]</sup> is pro-

posed as a means by which the dominance rule can be made applicable in more general cases. The model is based on the observation that human decision makers are cognitively unable to perform a multidimensional integration. Weak dominance states that if alternative  $A$  is at least as good as alternative  $B$  on all attributes, and alternative  $A$  is definitely better than alternative  $B$  on at least one attribute, then alternative  $A$  dominates alternative  $B$  (cf. Lee<sup>[10]</sup>; von Winterfeldt & Edwards<sup>[11]</sup>). The equate-to-differentiate model postulates that, in order to utilize the very intuitive or compelling rule of *weak dominance* to reach a binary choice between  $A$  and  $B$  in more general cases, the final decision is based on detecting  $A$  dominating  $B$  if there exists at least one  $j$  such that  $U_{Aj}(x_j) - U_{Bj}(x_j) > 0$  having subjectively treated all  $U_{Aj}(x_j) - U_{Bj}(x_j) < 0$  as  $U_{Aj}(x_j) - U_{Bj}(x_j) = 0$ , or, detecting  $B$  dominating  $A$  if there exists at least one  $j$  such that  $U_{Bj}(x_j) - U_{Aj}(x_j) > 0$  having subjectively treated all  $U_{Bj}(x_j) - U_{Aj}(x_j) < 0$  as  $U_{Bj}(x_j) - U_{Aj}(x_j) = 0$ , where  $x_j$  ( $j = 1, \dots, M$ ) is the objective value of each alternative on Dimension  $j$  (for an axiomatic analysis, see Li<sup>[6]</sup>). The principle of deciding which dimensional

difference is to be equated and which is to be differentiated is not the importance of the dimension (such as lexicographic rule<sup>[12]</sup>) but the intra-dimensional difference. The smaller differences of either insignificant dimension or important dimension will be equated thus leaving the greater dimensional difference to be differentiated as the determinant of the final choice.

The application of the equate-to-differentiate rule is straightforward, allowing the choice reversal phenomenon to be accounted for and to be predicted. The equate-to-differentiate view would be that in situations where there is variability in choice the alternative chosen on any one trial will be determined by the perceived greater dimensional difference on which the choice is based, and that there is variability in choice because the determinant dimensional difference changes. In other words, it is not the evaluation of the overall worth of the offered alternatives but the evaluation of the greatest one-dimensional difference between alternatives that is regarded as the cause which is most likely to be responsible for a tendency to reverse choice. Thus choice variability can be isolated as located in the change of determinant dimension. Such a "one-dimensional difference" account has been supported by using a "matching" task to examine whether the knowledge of the value difference of the paired outcomes along each dimension will permit prediction of choices in decision making under risk<sup>[13-15]</sup> and in decision making under uncertainty<sup>[9, 16]</sup>. These findings fit nicely with the equate-to-differentiate approach.

Instead of accounting for choice reversals by assuming that the utilities of the choice options are close (e.g., Leland<sup>[17]</sup>), the present research is based on the assumption that choice problems whose determinant dimensional differences are smaller would be most likely to produce choice reversals (i.e.,  $P(i, i, j) > 0.5$ ). Theoretically, problems with such characteristics exist throughout decision making under certainty, decision making under risk and decision making under uncertainty. Evidence of stochastic choice of alternative can be obtained by using a test-retest format but, on each test, matching information should permit prediction of the alternative chosen.

The aim of this research was to see whether the equate-to-differentiate approach could provide a possible

explanation and prediction for the observed choice reversals. The following choices across certainty, uncertainty and risk represent an attempt to carry out such a test.

## 2 Method

To accomplish this goal, three choice problems across certainty, uncertainty and risk were constructed so as to give rise to choice reversal data.

### 2.1 Riskless Choice

**2.1.1 Participants** The present questionnaire study was done with college students. Participants were 40 undergraduate students enrolled in General Psychology courses at Hwa Nan Women's College. They were unfamiliar with research on behavioral decision-making prior to the study and participated as volunteers.

**2.1.2 Materials and Procedure** The choice problem (Choice 1) used is riskless one, a choice between two university admissions. One is superior in the university offered while the other is superior in the speciality offered. The choice problem, coupled with a "matching" task where the outcomes of alternatives on each dimension are paired, was given to the participants. The choice and matching tasks are shown here exactly as they were posed to participants.

**CHOICE 1** Imagine that, as a candidate for the National Entrance Examination, you simultaneously received two admission notices after the examination, in which the universities and specialities to which you were admitted were as follows, which of them would you accept?

Admission A: To a local ordinary university under the jurisdiction of the Provincial Government; 2nd favourite speciality.

Admission B: To a local key university under the jurisdiction of the Provincial Government; 3rd favourite speciality.

Please circle your choice:                      A              B

*Matching* (Circle the one whose alternatives are most different)

C. "local ordinary university" vs "local key university"

D. "2nd favourite speciality" vs "3rd favourite speciality"

The choice and matching tasks were given twice to the same student participants in booklets with an interval of 79 days. When the completed questionnaires were collected, the participants were then debriefed.

## 2.2 Choice under Uncertainty

**2.2.1 Participants** A total of 29 volunteer undergraduate students at the Department of Psychology at Zhejiang University participated in this questionnaire study.

**2.2.2 Materials and Procedure** The second choice problem (Choice 2) represents choice under uncertainty. It is a modified version of a choice problem reported by Li<sup>[16]</sup>. In the original choice problem, the alternative chosen most often (71%) is the sure gain of \$25\*. In order to make the choice more even, the sure gain is reduced from 25 in the original to 15 in the present one. This is based on the consideration that the judged difference between "a sure gain of

15" and "an unknown chance to gain nothing" should be smaller than that between "a sure gain of \$25" and "an unknown chance to gain nothing", thus leading to choosing uncertainty option (choosing the option with the greater best possible outcome having treated the worst possible outcomes as subjectively equal). The choice and matching tasks read as follows:

### CHOICE 2

*Choice* (Circle the alternative you would prefer to have)

A. A sure gain of 15.

B. An unknown chance to gain an unknown amount of money more than 15 or to gain nothing

*Matching* (Circle the one whose alternatives are most different)

C. "A sure gain of 15" vs "An unknown chance to gain an unknown amount of money more than 15"

D. "A sure gain of 15" vs "An unknown chance to gain nothing"

The choice and matching tasks were given twice to the same group of participants in booklets with an interval of 39 days.

## 2.3 Choice under Risk

**2.3.1 Participants** The participants were 27 vol-

unteers who were senior executives working at the Bank of China, Fujian Branch.

**2.3.2 Materials and Procedure** The third choice problem is one under risk. In this choice (Choice 3), individuals are presented with two gambles, one featuring a high probability of winning a modest sum of money, the other featuring a low probability of winning a larger amount of money. According to Kahneman and Tversky<sup>[18]</sup>, most people will choose the gamble in which winning is more probable (0.80), that is, Alternative A. On the other hand, the equate-to-differentiate model suggests that decreasing the value of the payoffs will result in a greater overall preference for Alternative B, the one which offers the larger prize but in which winning is not probable (0.40). Therefore, when the gamble probabilities meet prospect theory's postulate but the gamble payoffs meet that of the equate-to-differentiate model, the offered alternatives would be expected to be equally attractive. Booklets, which contained the following choice and matching tasks, were administered to the participants twice with an interval of 63 days.

### CHOICE 3

*Choice* (Circle the alternative you would prefer to have)

A. You have a 80% chance of getting 30, but a 20% chance of getting nothing

B. You have a 40% chance of getting 60, but a 60% chance of getting nothing

*Matching* (Circle the one whose alternatives are most different)

C. "80% to win 30" vs "40% to win 60"

D. "20% to win nothing" vs "60% to win nothing"

## 3 Results and Discussion

The overall statistical results are summarized in Table 1. It can be seen that Choice 1 is constructed so that Admission A is better than Admission B on the "speciality" dimension while Admission B is better than Admission A on the "university" dimension. It is reasoned by the equate-to-differentiate's one-dimensional difference account that if the participant thinks that one of the two pairs is the "most equivalent" according to his or her utilities, he or she will choose the alterna-

\* The results reported by Li<sup>[16]</sup> were that most of the Australian participants (71%) avoided the uncertain option. Coupled with this was the fact that most (90%) of those who preferred the sure gain chose the matched pair D, "a sure gain of \$25" vs "an unknown chance to gain nothing", as the most different one. An analysis reveals that matching significantly accounted for 42% ( $\phi^2$ ,  $p < 0.01$ ) of the choice variance in this choice problem.

tive with the better outcome in the "most different" pair That is, participants who selected Admission A (or B), tend to base their final choice on only the "speciality" (or "university") dimension, having trea-

ted the values on the "university" (or "speciality") dimension as if they were equal To express this operationally, if C (D) is circled most different then B (A) will then be chosen, and vice versa

Table 1 Statistical data for the test-retest results

No. of choice problem	N	I	r	$\phi_1$	$\phi_2$	$R^2$
1	40	79	0.23	-0.30*	-0.34*	0.12*
2	29	39	0.25	-0.40*	-0.40*	0.27**
3	27	63	0.32	-0.47**	-0.54**	0.34**

Note N = number of participant used; I = test-retest interval in days; r = test-retest correlation (a reliability coefficient) (from Table 4);  $\phi_1$  = phi at the first test (from Table 2);  $\phi_2$  = phi at the second test (from Table 3);  $R^2$  = the proportion of variance in changing choice accounted by changing matching (from Table 5).

In the light of a representation system (with the best possible and the worst possible outcome dimensions) to describe both Choices 2 (as shown in Figure 1) and 3, Alternative A is seen as better than Alternative B on the worst possible outcome dimension while Alternative B is seen as better than Alternative A on the best possible outcome dimension, assuming that what people ultimately wanted in hand is an amount to win but not a chance of winning It is anticipated by the equate-to-differentiate model that, in order to utilize weak dominance to reach a decision, people have to "equate" smaller difference between options on either the best possible or the worst possible outcome dimension, thus leaving the greater one-dimensional

difference to be differentiated as the determinant of the final choice That is, if Alternative A is chosen, the participant should choose the pair of two "worst possible outcomes" (D) as most different, thus leading to an "aim to avoid the worst" process On the other hand, if Alternative B is chosen then the participant should choose the pair of two "best possible outcomes" (C) as most different, thus leading to an "aim for the best" process

The observed results of these equate-to-differentiate predictions across all the three choice problems from the first and second trial are shown in Tables 2 ~ Table 3 respectively.

Table 2 Choice and matching data from the first trial in Choices 1 ~ 3

		CHOICE 1		CHOICE 2		CHOICE 3	
		Choice		Choice		Choice	
		A	B	A	B	A	B
Matching	C	6	(14)	2	(13)	3	(8)
	D	(12)	8	(7)	7	(12)	4

Note The data in brackets are numbers of respondents who chose according to the equate-to-differentiate model

Table 3 Choice and matching data from the second trial in Choices 1 ~ 3

		CHOICE 1		CHOICE 2		CHOICE 3	
		Choice		Choice		Choice	
		A	B	A	B	A	B
Matching	C	5	(17)	1	(12)	3	(8)
	D	(10)	8	(7)	9	(13)	3

Note The data in brackets are numbers of respondents who chose according to the equate-to-differentiate model

This result was replicated with 32 student participants from the School of Psychology at the University of New South Wales ( $\phi = -0.41, p < 0.02$ ). Alternative A in Choice 2, the sure thing option, can itself be seen as either the best possible outcome (when compared with the best possible outcome of the uncertain option) or the worst possible outcome (when compared with the worst possible outcome of the uncertain option). Such a representation and hence a manipulation of risky preference involving a sure thing option is empirically tested in Li's study (e.g., Li<sup>[5, 15, 19, 20]</sup>).

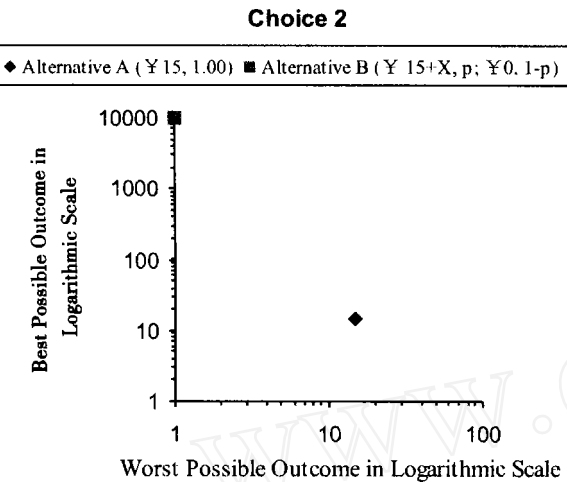


Fig. 1 The representation of Choice 2 by applying a logarithmic utility function

An analysis of the contingency tables reveals that the relevant  $\phi$  (phi) coefficients relating choice and matching for both the first and second tests were significant, falling between 0.30 and 0.54, with a mean of 0.41. In particular, there is a pretty large effect (eta squared) of the "matching" of paired outcomes on choice, that is, matching significantly accounted for 9.0%, 16.0% and 22.1% of the choice variance in Choice 1, Choice 2 and Choice 3 from the first trial and 11.6%, 16.0% and 29.2% of the choice variance in Choice 1, Choice 2 and Choice 3 from the second trial respectively. The present equate-to-differentiate model does not assume that the

individual is able to perform a utility-integration calculation, and instead holds that when dominance does not exist, the choice then has to be made according to subjective dominance detecting rather than any kind of overall maximizing. The explanatory mechanism provided by the equate-to-differentiate model is a coherent one across the three decision domains. Taken together, knowledge of paired "most different" outcomes chosen by participants does permit a satisfactory explanation or prediction of the observed choice preferences. Such a finding, together with those obtained in other decision problems<sup>[8, 9, 13-15]</sup>, adds to evidence pointing to fundamental limitations in people's capacity to process information.

On the other hand, the 3 choice problems test-retest reliabilities fell between 0.23 and 0.32 in an average 60-day interval (see Table 4). None of them is significant. A number of participants' choices (37.5% in Choice 1, 31% in Choice 2 and 33% in Choice 3) changed after the test-retest interval (see Table 4). The generally low reliability confirms the present prediction that stochastic choice of alternative can be obtained by using a test-retest format and that choice is not deterministic but probabilistic. The inconsistencies observed will pose greater challenges for conventional choice models to cope effectively with these difficulties.

Table 4 A contingency table for the test-retest data to indicate choice consistency in Choices 1 ~ 3

		CHOICE 1		CHOICE 2		CHOICE 3	
		Trial 1		Trial 1		Trial 1	
		A	B	A	B	A	B
Trial 2	A	(9)	6	(4)	4	(11)	5
	B	9	(16)	5	(16)	4	(7)

Note: The data in brackets are the numbers of respondents who made consistent choices across the first and second trials

The most relevant finding (see Table 5) is that the change of choice can be accounted for by the change of matching. The effect size (proportion of variance accounted for) is 0.12, 0.27 and 0.34 in Choice 1, Choice 2 and Choice 3 and is significant for all the three domains of choices. Thus, the results support the notion that participants do not adopt different decision rules in their repeated choices. It seems that the resulting inconsistent responses in all three domains

of choice can be reasonably accounted for by the equate-to-differentiate rule in a consistent way. It is therefore expected that choice reversals in repeated measurement and other perplexing paradoxical patterns of behavior should be observed in fact when people's equate-to-differentiate strategy (deciding which dimensional difference is to be equated and which is to be differentiated) is caused to change.

Table 5 A contingency table for the test-retest data to indicate choice consistency and matching consistency in Choices 1 ~ 3

		CHOICE 1		CHOICE 2		CHOICE 3	
		Choice		Choice		Choice	
		V	U	V	U	V	U
Matching	V	(8)	5	(6)	3	(8)	5
	U	7	(20)	3	(17)	1	(13)

Note V = respondents whose choices or matching were varied across the first and second trials; U = respondents whose choices or matching were unvaried across the first and second trials. The data in brackets are the numbers of respondents whose choice strategy co-varied with their matching strategy.

4 Concluding Remarks

In sum, existing psychological models are successful only when considering crude measures of fit, such as the overall percentage of correct predictions or explanations based on randomly chosen stimuli. They fail to describe two very basic facts about human decision-making behavior—the variability and the temporal evolution of preferences. Even when results are highly significant, previous theories predict only modal responses, with no systematic accounting for minority responses.

The present experimental results are of interest because they account for temporal features of the deliberation process and suggest that observed choices as well as choice reversals are systematic, consistent, and predictable, and that this is so without resort to an ad hoc assumption that the probability of choosing one alternative over another is an increasing function of the overall utility of the alternative. If there is to be a model that can account for the large individual differences and for minority responses then the equate-to-differentiate model is a plausible candidate.

The present study suffers from some limitations. First, the test-retest interval across certainty, uncertainty and risk was varied from 39 to 79 days, which might cause some difficulty in comparing the variability of choices. Second, only one choice problem was designed and tested for each of these three domains. It appears that a further study evaluating various choice problems in each decision domain might be worthwhile. These features of the present study arouse some concerns regarding the external validity of the findings.

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确定、不确定及风险状态下选择反转:“齐当别 选择方式的解释

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摘 要 应用广义“弱优势”(weak dominance)模型检验确定、不确定及风险状态下的选择反转现象。该模型将人们的二择一选择行为描述为一种搜寻一备择方案在主观上优越于另一备择方案的过程。即:在甲方案在某一维度上优越于乙方案,而乙方案在另一维度上优越于甲方案的情况下,为了利用“弱优势”(weak dominance)原则达成决策,人们必须在一维度上将两者间较小的差异人为地“齐同”掉,而在另一维度上将“辨别”两者间较大的差异作为最终选择的依据。因此,在每次选择时,如果不认为最大的差异都是来自同一维度,就会导致选择反转。此项研究设计了一“匹配”任务,并借此检验,在不同的决策状态下,判断两备择方案在各维度上的差异是否能预测人们的重复选择变异。总的测试-再测试结果支持“齐当别”选择方式的解释。其发现表明:重复选择之所以可能是一致的,并不是因为每次都认定被选中的备择方案具有最大值,而是因为每次选择都认定最大的差异来自一固定的维度。

关键词 重复选择,选择反转,弱优势原则。

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